Squeezing a Mixed State

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Introduction

Spin squeezing can be used to reduce the atomic projection noise which is currently a limiting factor in fountain-type Ramsey spectroscopy [1, 2].

Despite the fact that most realistic state preparations, e.g. optical pumping, have some imperfections, a pure initial state is often assumed to simplify the analysis of spin squeezing experiments. We present an analysis of how initial state imperfections influence the expected experimental outcome.

Modelling an optically pumped ensemble

We model an optically pumped ensemble as a collection of uncorrelated, identically distributed atoms. The individual atoms are described by their pseudospin equivalent, $\boldsymbol{S}$, and are in the two-level case assumed to be in an incoherent mixture of their $|\uparrow\rangle$ and $|\downarrow\rangle$ states with populations p and $1-p$ respectively.

To describe spin squeezing we are only interested in the collective atomic pseudospin, $\boldsymbol{J} = \sum_i \boldsymbol{S}_i$, of the ensemble. Our analysis show that

- The ensemble is in an incoherent mixture of simultaneous $\boldsymbol{J}^2$ and J_z eigenstates, $\left| \begin{smallmatrix} j \\ m \end{smallmatrix} \right\rangle$.
- The population of the $\left| \begin{smallmatrix} j \\ m \end{smallmatrix} \right\rangle$ state is

$$p^{(n)}_{j,m} = \frac{2j+1}{j_0+j+1} \binom{2j_0}{j_0+j} p^{j_0+m}(1-p)^{j_0-m}, \tag{1}$$

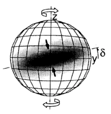
with $n = 2j_0$ being the number of atoms.

The distribution is discussed in the box *Dicke state population* in the right margin.

Squeezing a mixed state

We have investigated how an atomic ensemble described by Eq. (1) evolves under two unitary squeezing operations.

One-axis twisting



A Hamiltonian proportional to J_x^2 will squeeze a spin aligned along the z-axis [3][a]. Since the direction of the squeeze axis, as well as the optimal interaction time, depend on j, this squeezing method will not perform optimally for a mixed state.

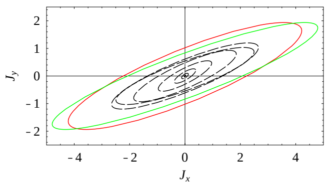
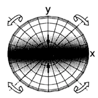


Fig. 1. The result of squeezing 15 atoms initially 90% polarized ($p = 0.9$). The red ellipse represent the distribution of the transverse spin components. The green ellipse shows how this would have looked in the case of an initially perfectly polarized ensemble. The black ellipses illustrate the contribution from the different $\boldsymbol{J}^2$ eigenspaces.

[a] The illustration shows a J_z^2 squeezing operation.

For an ensemble described by Eq. (1) we find, however, that the spread in rotation angle and optimal interaction time over the populated j-values is very small. This leads us to believe that one-axis twisting should function well even with a mixed state, as supported by Fig. 1.

Two-axis countertwisting



By simultaneously twisting around two axes, as by means of a Hamiltonian proportional to $J_+^2 - J_-^2$, we achieve a fixed squeezing axis.

In this case we have obtained a quantitative description of the impact of imperfect state preparation in terms of the influence on the squeezing parameter, as defined in Eq. (4). We find that to lowest order in $1-p$ the corrected squeezing parameter is

$$\xi \approx \xi_0 \left[1 - 2(1-p)\log \xi_0\right], \tag{2}$$

where ξ_0 is the squeezing parameter that would have been obtained with a perfectly polarized ensemble.

Conclusion and outlook

We have found that squeezing by one-axis twisting and two-axis countertwisting is quite stable with respect to imperfectly polarized initial states.

The results presented here are derived in Ref. [4]. Here we also discuss the extension to atoms with more than two levels, where the ensemble shows coherences between different $\boldsymbol{J}^2$ eigenspaces.

The approach used here is readily extendible to other areas. At present we are studying entanglement of two atomic ensembles as reported by Julsgaard *et.al.* [5]. Another possible continuation of the work is to analyze the effect of imperfect initial state preparation on squeezing methods based on quantum non-demolition measurements [6, 7].

Acknowledgments

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Spin squeezing in precision spectroscopy

An obvious way to determine if the atoms of an atomic ensemble are in a precise superposition of $|\uparrow\rangle$ and $|\downarrow\rangle$ is to count the number of atoms in the two states.

The $\sqrt{n}$ counting noise associated with this measurement may be considered as the projection noise associated with measuring the J_z component of the collective pseudospin. Spin squeezing reduces the projection noise without seriously influencing the mean spin, thus leaving the state well suited for spectroscopy.

As an example, Fig. 2 illustrates how spin squeezing can be used to enhance the precision of Ramsey spectroscopy by reducing the uncertainty on J_z:

$$J_z = \frac{\hbar}{2}\left(n_\uparrow - n_\downarrow\right), \tag{3}$$

which is the population difference measured at the end of the Ramsey process. The improvement of spectroscopic precision caused by the spin squeezing is quantified by the squeezing parameter, ξ: the ratio between the squeezed and unsqueezed precisions:

$$\xi = \frac{\Delta\omega_{\text{squeezed}}}{\Delta\omega} \tag{4}$$

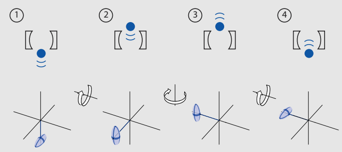

Fig. 2. The evolution of the collective pseudospin of an atomic ensemble during a fountain-type Ramsey spectroscopy experiment: After being trapped and cooled in a magneto-optical trap, the ensemble is launched upwards through an RF cavity. After a period of free fall, the ensemble eventually falls back through the same cavity.

Dicke state population

An interesting point illustrated by the distribution (1) is that the symmetric Dicke states, $\left| \begin{smallmatrix} j_0 \\ m \end{smallmatrix} \right\rangle$, which are the only states populated in a perfectly polarized ensemble, are hardly populated in realistic applications. In fact, we find that for the $j = j_0$ eigenspace to be the most populated, the expected number of atoms in the $|\downarrow\rangle$ state has to be less than one, corresponding to $(1-p)n < 1$.

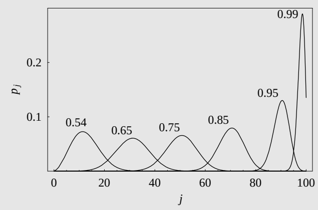

Fig. 3. The distribution in j of an ensemble of 200 two-level atoms at various degrees of polarization as quantified by the p-values printed above the peaks.